Numerical Methods, project C, Number 32

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Chapter 1

Determine polynomial function fitting experimental data

1.1 Problem

Given following samples:

x_i	y_i
-5	-6.5743
-4	0.9765
-3	3.1026
-2	1.8572
-1	1.3165
0	-0.6144
1	0.1032
2	0.3729
3	2.5327
4	7.3857
5	9.4892

We have to determine polynomial function $y = f(x)$ that best fits this data.
We will use least-square approximation using system of normal equation with QR factorization.

1.2 Theoretical introduction

We have polynomial function:

$$y(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0$$

where n - degree of the polynomial we use to approximate function
Given matrix $\mathbf{A} \in \mathbb{R}^{m \times n} \quad (m > n)$ and vector $\mathbf{y} \in \mathbb{R}^m$
We need to find vector x̂ such that:

$$\forall x \in \mathbb{R}^n \quad ||\mathbf{y} - \mathbf{A}\hat{x}||_2 \leq ||\mathbf{y} - \mathbf{A}x||_2$$

Chapter 2

Determine trajectory of the motion

2.1 a) Runge-Kutta method of 4^{th} order and Adams PC

2.1.1 Problem

We are given following equations:

$$\frac{dx_1}{dt} = x_2 + x_1(0.5 - x_1^2 - x_2^2)$$

$$\frac{dx_2}{dt} = -x_1 + x_2(0.5 - x_1^2 - x_2^2)$$

And we have to determine the trajectory of the motion on interval $[0, 15]$ with following initial conditions: $x_1(0) = 8; x_2(0) = 9$ In this section we will use Runge-Kutta method of 4^{th} order and Adams PC with different step-sizes until we find an optimal constant step size - when the decrease of the step size does not influence the solution significantly.

2.1.2 Theoretical Introdution

2.2 b) Runge-Kutta method of 4^{th} order with variable step size automatically adjusted

2.2.1 Problem

We are given following equations:

$$\frac{dx_1}{dt} = x_2 + x_1(0.5 - x_1^2 - x_2^2)$$

$$\frac{dx_2}{dt} = -x_1 + x_2(0.5 - x_1^2 - x_2^2)$$

And we have to determine the trajectory of the motion on interval $[0, 15]$ with following initial conditions: $x_1(0) = 8; x_2(0) = 9$ In this section we will use Runge-Kutta method of 4^{th} order with step size automatically adjusted by the algorithm, with error estimation made according to the step-doubling rule.

2.2.2 Theoretical Introdution

Bibliography

[1] Piotr Tatjewski (2014) *Numerical Methods*, Oficyna Wydawnicza Politechniki Warszawskiej